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Teaching strategies in the learning of highest common factor and lowest common multiple

N L A Halim[1], H-C Li[1], M Shahrill[1] and R C I Prahmana[2]

[1] Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam, Bandar Seri Begawan, Brunei Darussalam
[2] Universitas Ahmad Dahlan, Jl. Pramuka Kav. 5, Pandeyan, Yogyakarta, Indonesia

E-mail: masitah.shahrill@ubd.edu.bn

Abstract. The main purpose of this study is to improve students' understanding of Highest Common Factor (HCF) and Lowest Common Multiple (LCM). Several alternative teaching strategies were integrated in the Year 7 lessons involving 20 students from one secondary school in Brunei Darussalam. The categories identified in the teaching strategy were the application of group work, embedding real-life problems, using presentations and the traditional drilling practice method. An open-ended survey was disseminated to collate the students' feedback, and among the questions posed was in relation to the different types of lesson activities utilised within each of the teaching strategies. Although 55% of the students preferred the traditional drilling practice on the calculation of HCF and LCM, mainly due to the ease of the task as opposed to activities involving real-life problems, the findings also indicated that most students favour sharing their ideas in a healthy competition learning style between each other or between the groups.

1. Introduction

Learning Highest Common Factor (HCF) and Lowest Common Multiple (LCM) have several points of importance, such as calculating when two events will occur at the same time given different recurring periods, and to group two or more sets of items into their largest possible amount. One of the difficulties that students may have when solving real-life problems involving HCF and LCM, is the implication of grasping the key concepts and understanding the problem and the solution [1, 2]. The difficulties in understanding the concepts of LCM are based on the teaching practices at the formal level that are often regarded as procedural and manipulative [2]. Previous studies have reported that the meaning of HCF and LCM often confuses many students [2, 3]. To begin with, students need to have an understanding of the terms 'a factor' and 'a multiple'. For example, 4 x 8 = 32, 4 and 8 are factors of 32, and 32 is a multiple of 4 and 8. According to the Year 7 mathematics textbook currently being used in Brunei, the students are instructed to solve HCF and LCM by using the prime factorisation method or the continuous division method [4]. Heng [4] defined the HCF as "the Highest Common Factor of two or more numbers is the largest common factor of these numbers" (p 8) while "a common factor for two or more numbers is a factor that is common to all numbers" (p 8). Meanwhile, the Lowest Common Multiple of two or more numbers is defined as "the smallest common multiple of these numbers" [4, p 9]. The LCM can be found by listing out the multiples of numbers and by choosing the smallest numbers among the common multiples.

1.1. Perspectives involving the teaching and learning of HCF and LCM

In a previous study from Brunei, a number of students were asked to find the HCF of 12 and 24, and very few could obtain the solution answer of 12 through deduction, having realised that 12 is a factor of 24 [5]. Suffolk [5] added that a lot of schools did not even teach the basic meaning of HCF and LCM. When the question "*What is the highest common factor of 12 and 16*?" was posed to the students, the teachers themselves deemed the question unreasonable as they only taught the students how to solve it and not what it meant. Suffolk [5] concluded that this method of doing without thinking could have severe consequences towards the students' learning capabilities, not just within the topic of HCF and LCM, but towards learning other topics in mathematics as well. In fact, there is a shortcut when calculating HCF and LCM with the use of either continuous division or factor tree of prime factorisation method, which is by using a larger divisor, instead of using prime number as the divisor, for the two or more numbers [5]. However, he expressed that students were using the shortcut method without prior learning to the reason why the lowest prime factor needed to be used first as the divisor for the numbers. He also argued that while students appeared to know how to use these methods of prime, there was no afterthought towards the answer or any discussions regarding it, and it was unclear whether the students really understood the concept that underlies the operation.

In a different study involving 248 students from Pakistan, Mohyuddin and Khalil [3] identified potential misconceptions experienced by the primary school students in the learning of mathematics involving HCF and LCM. One of the questions posed was to calculate the LCM of 12 and 18. They found that one-third (77 students) answered correctly (the answer is 36), and almost two-thirds responded using the HCF method, which indicated that the students might be confused between HCF and LCM. Furthermore, they found that 42 students thought that the number 18 was the LCM while the rest could not answer the question. These results suggest that the concept of HCF and LCM has yet to be fully understood [3]. The other question that they gave was about real-life problem question that is, "*A class was divided into two groups of children. In one group there were 16 children whereas 12 were in the other group. Minimum how many apples are needed so that if divided in the first or the second group, the apples are divided completely?*" [3, p 142]. This question required the students to apply their understanding of LCM. The results obtained were 33% of the students gave the correct answer, while 32% picked the answer that would have been the answer to the HCF method, and the rest were unable to answer this question. Statistically, disregarding the students who did not provide an answer, there was a 50-50 distribution of students using methods to solve either using HCF or LCM. This suggests that when attempting a question, the students were only able to relate the question to HCF or LCM if the problem question was stated clearly, as portrayed in the first question. However, when faced with a worded problem without clearly stating the use of HCF or LCM, the students had an inability to relate the question to the term LCM due to a lack of understanding [3].

1.2. The strategies used in teaching

Real-life applications are essential to help students acquire logical thinking and to show them the relevance of mathematical concepts throughout their learning [6]. In addition to the explanations, along with the aid of diagrams, pictures or graphs may help provide the students with clear conceptual understanding. A significant increase in students' understanding and achievements could occur with the application of real-life examples in problem solving [7-9]. Allowing students to portray their own ideas on real-life application may help enhance their understanding and level of interest on the subject [7]. Meanwhile, Triyani et al. [2] used the story telling approach of the Legend *Putri Dayang Merindu* (LPDM) as their teaching strategy. They identified the LPDM's role in facilitating students' capacity to improve conceptual understanding in LCM by solving situational problems based on the LPDM and eventually moving to formal solution of LCM. Accordingly, previous studies have shown positive effects by using story telling in order to achieve meaningful and enjoyable experiences in the learning of mathematics [2, 10-12].

Khatoon and Akhter [13] designed a teaching strategy using innovative collaboration along with group learning to aid the learning of slow learners. The results suggested that through the use of this

teaching strategy, there was significant improvement in the performances of slow learners when comparisons were made of their midterm and final term results. In addition, the students in their study showed increased involvement when given the opportunity to work with their peers. Having students working in groups allows students to initiate discussions, deepen their interactions with the subject and develop critical thought processes [13-15]. Johnsen [16] examined the effects of group work on students when establishing mathematical understanding and achievements. The study involved 13 students subdivided into two groups. One group applied the teacher-based learning while the other group centred on student-based learning. The overall results were that the students improved relations between themselves and worked better as a group rather than individually. Johnsen [16] concluded that the group performed much better if they worked collectively for a longer period of time.

Having students to give presentations of their work has proven to be significant as it enhances their communication and public speaking skills [17-19]. Sharing mathematical ideas through presentations, and the application of grouped or paired work provides students the opportunity to discuss and share ideas with one another, to improve on their task, and to provide and receive constructive criticisms on the existing ideas [19]. Consequently, partaking in discussions with the entire class also promotes the opportunities to listen and challenge other peers' ideas [19].

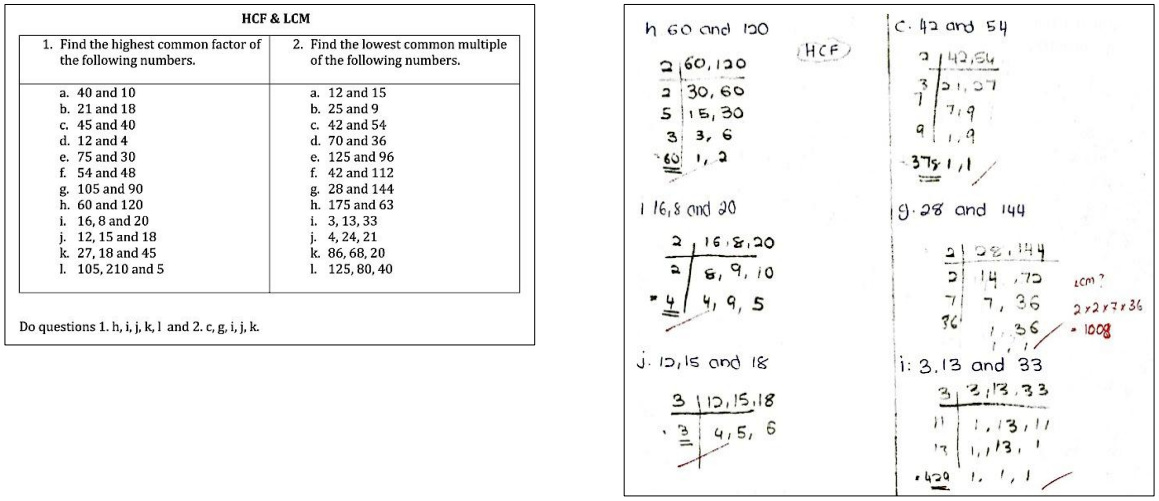
2. Methodology

The main purpose of this study is to improve students' understanding of HCF and LCM. It also aims to investigate the effects of using HCF and LCM questions based on real-life situations to further facilitate the students' understanding of the topic. Integration of real-life situations into the prospect of HCF and LCM may allow the students to relate to the question on a personal level, instead of undergoing a step-by-step process of solving the equations. This present study involves teaching HCF and LCM by implementing several alternative teaching strategies through a series of intervention lessons using an action research framework.

2.1. Participants and intervention design

The participants in this present study consisted of 20 students (5 males and 15 females) from a Year 7 class in one of the secondary schools in the Brunei-Muara District. All the relevant permissions from the school, teachers, parents and the students were obtained before initialising this study. The first author took on the role as the teacher and conducted a total of five lessons during the course of the intervention. Each lesson lasted about 50 minutes and the lessons consisted of revision extensions, drilling practices, learning activities, pair work, group work, problem-based learning and presentations. Descriptions of each of the intervention lessons are described as follows.

The first lesson of the intervention focused upon the methods and calculation to solve HCF and LCM. The students were provided with a list of direct calculations of the HCF and LCM problems, and they were initially instructed to work out the solutions on their own. Subsequently, the students were paired to let them check each other's answers while having discussions between them. Finally, the solutions were discussed together on the whiteboard with the opportunity for other students in the class to ask questions. The type of work involved is pair work and the activities involved recalling the definitions of, and the drilling practice on the calculations of HCF and LCM. Figure 1(a) represents the exercise questions and Figure 1(b) shows a sample of a student's work in the first lesson.



HCF & LCM

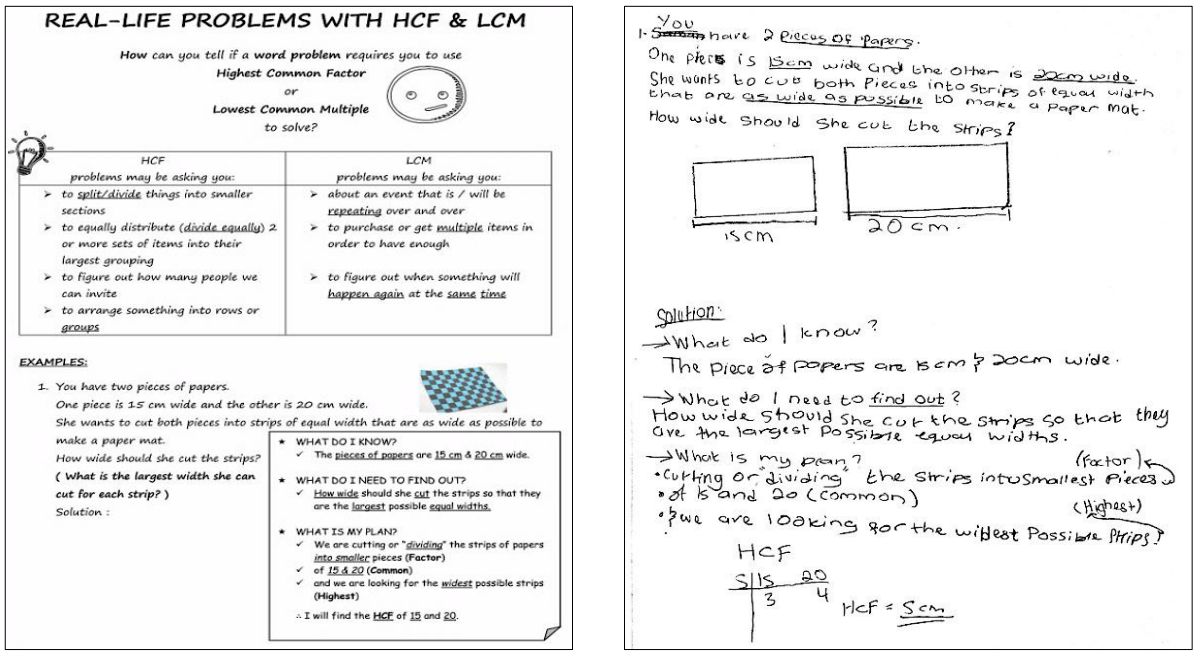
1. Find the highest common factor of the following numbers.	2. Find the lowest common multiple of the following numbers.
a. 40 and 10	a. 12 and 15
b. 21 and 18	b. 25 and 9
c. 45 and 40	c. 42 and 54
d. 12 and 4	d. 70 and 36
e. 75 and 30	e. 125 and 96
f. 54 and 48	f. 42 and 112
g. 105 and 90	g. 28 and 144
h. 60 and 120	h. 175 and 63
i. 16, 8 and 20	i. 3, 13, 33
j. 12, 15 and 18	j. 4, 24, 21
k. 27, 18 and 45	k. 86, 68, 20
l. 105, 210 and 5	l. 125, 80, 40

Do questions 1. h, i, j, k, l and 2. c, g, i, j, k.

(a) (b)

Figure 1. Samples of questions and a student's work from Lesson 1

The second lesson integrated the application of real-life problems in HCF and LCM. The students were also taught to identify hints within the worded problems so as to assess the nature of the question on whether it was HCF or LCM. The students were provided with a few practice questions (Figure 2(a)) to understand more on the topic with the help of teaching aids, such as accurate measurements of paper mats and number of sweets to help them visualise the problem and practically experience the problem. The students were given the chance to try solving the questions by working in groups. Figure 2(b) represents a sample of a student's work in the second lesson.



REAL-LIFE PROBLEMS WITH HCF & LCM

How can you tell if a **word problem** requires you to use
Highest Common Factor
or
Lowest Common Multiple
to solve?

HCF problems may be asking you:	LCM problems may be asking you:
➢ to split/divide things into smaller sections	➢ about an event that is / will be repeating over and over
➢ to equally distribute (divide equally) 2 or more sets of items into their largest grouping	➢ to purchase or get multiple items in order to have enough
➢ to figure out how many people we can invite	➢ to figure out when something will happen again at the same time
➢ to arrange something into rows or groups	

EXAMPLES:

1. You have two pieces of papers.
One piece is 15 cm wide and the other is 20 cm wide.
She wants to cut both pieces into strips of equal width that are as wide as possible to make a paper mat.
How wide should she cut the strips?
(What is the largest width she can cut for each strip?)
Solution :

★ WHAT DO I KNOW?
✓ The pieces of papers are 15 cm & 20 cm wide.

★ WHAT DO I NEED TO FIND OUT?
✓ How wide should she cut the strips so that they are the largest possible equal widths.

★ WHAT IS MY PLAN?
✓ We are cutting or "dividing" the strips of papers into smaller pieces (Factor)
✓ of 15 & 20 (Common)
✓ and we are looking for the widest possible strips (Highest)

∴ I will find the HCF of 15 and 20.

1. You have 2 pieces of papers.
One piece is 15cm wide and the other is 20cm wide.
She wants to cut both pieces into strips of equal width that are as wide as possible to make a paper mat.
How wide should she cut the strips?

15cm 20 cm.

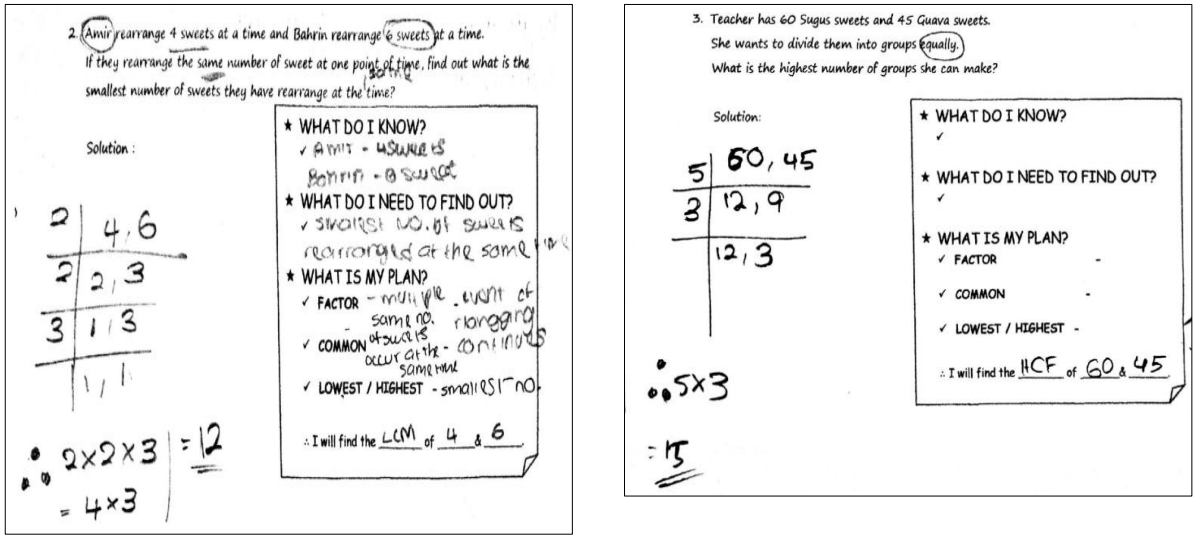
Solution:
→What do I know?
The piece of papers are 15cm & 20cm wide.
→What do I need to find out?
How wide should she cut the strips so that they are the largest possible equal widths.
→What is my plan?
•Cutting or dividing the strips into Smallest Pieces (Factor)
• of 15 and 20 (common)
• We are looking for the widest possible strips (Highest)

HCF
5 | 15 20
3 4
HCF = 5cm

(a) (b)

Figure 2. Samples of questions and a student's work from Lesson 2

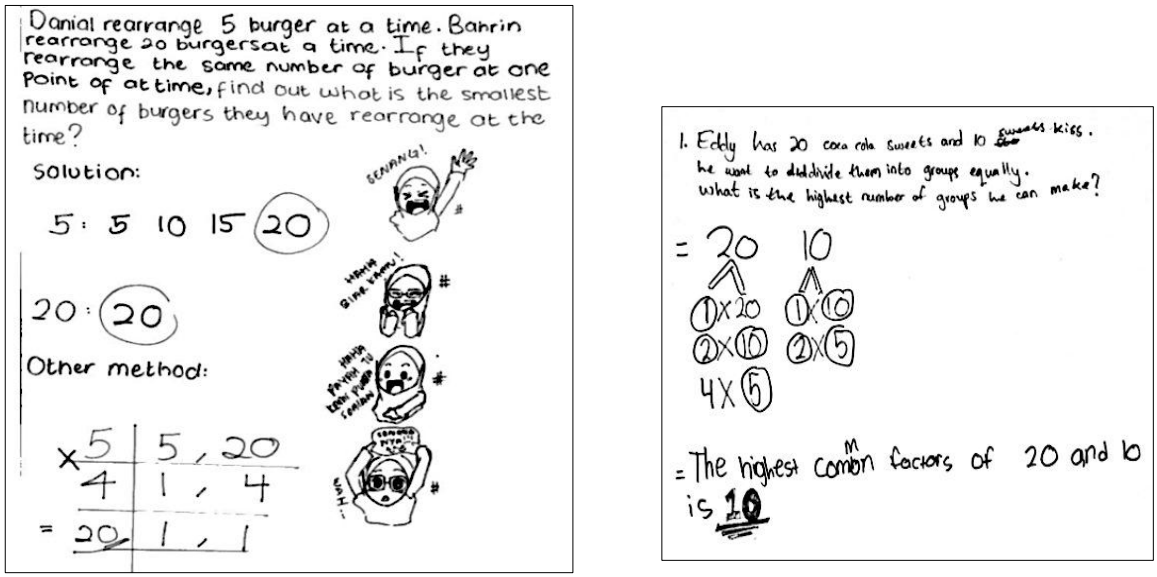
In the third lesson, the students continued solving the questions from the previous lesson (shown in Figures 3(a) and 3(b)). Additionally, each group had to discuss and create a question relating to real-life application of HCF and LCM for the other groups to answer in the following lesson.



(a) (b)

Figure 3. Samples a student's work from Lesson 3

The fourth lesson involved distributing the questions created from the third lesson to the other groups other than their own. The students were allocated time to discuss and answer the question. Each group had to prepare their responses on a blank piece of A3 sized paper in order to present them in the next lesson. Selected samples of the group work are provided in Figures 4(a) and (b).

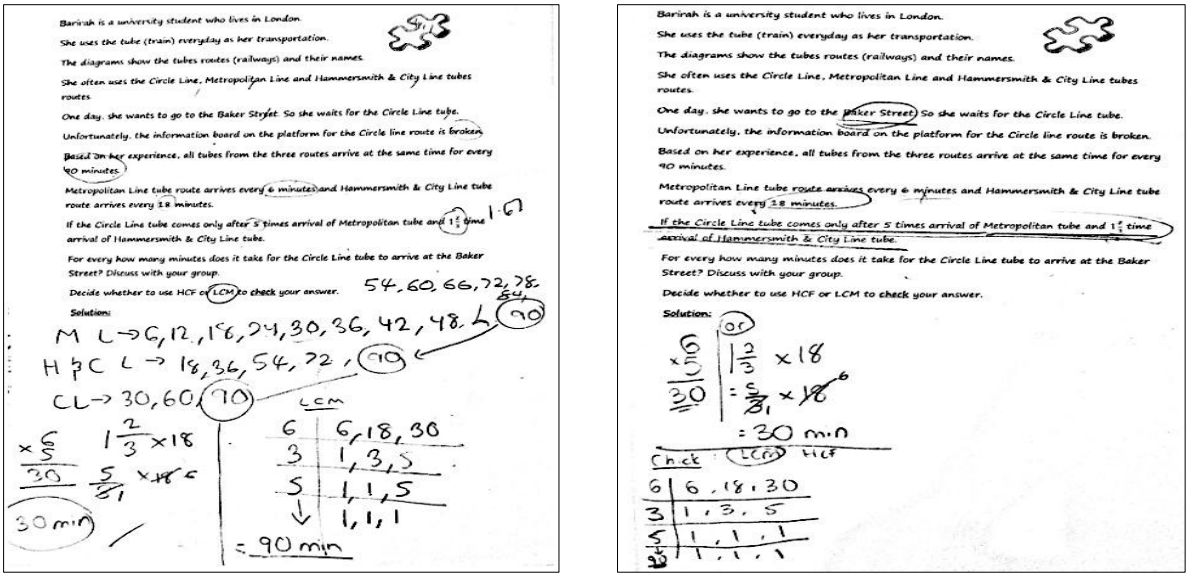


(a) (b)

Figure 4. Selected samples of the group work from Lesson 4

In the final lesson, each of the 6 groups presented their work with the solutions explained by one of their nominated group members (refer to sampled works in Figure 4). Each group that presented were also enquired if the correct methods were used to solve the problems given to them. They also had the opportunity to check with the respective creators of the questions for verification of the worked solutions. This activity allowed all the students to gain and share ideas within the allocated discussion time. The final task given to the class was a question that involved problem-based learning on solving real-life problems using HCF and LCM. This question required higher-order thinking skills in order to

solve it. Once the group task was completed, discussions were done together with the teacher's guidance. Displayed in Figures 5(a) and (b) are the selected samples of the group work from Lesson 5.



(a) (b)

Figure 5. Selected samples of the group work from Lesson 5

2.2. Data collection and analysis

An open-ended survey was distributed to gather the students' feedback. The survey assessed the students' attitudes towards the intervention and their level of understanding to real-life problems in HCF and LCM. The students were given 30 minutes to complete the survey. The collected data were analysed quantitatively and qualitatively through the students' responses from the survey. Consequently, the results reported in this paper will only focus on the selected questions presented in the survey that were, firstly, related to the different categories of lesson activities employed within each of the teaching strategies during the intervention lessons; secondly, recalling the keywords for HCF and LCM; and thirdly, extracting suggestions for improvements in solving real-life problems involving HCF and LCM.

3. Results

First to be presented here are the students' feedbacks regarding the five lesson activities summarised in Table 1. For this survey question, the students were required to rate from 1 (the most favourite) to 5 (the least favourite) from the series of lesson activities and also to state the reasoning for their choices. The results of the students' rating for each of the five activities are shown in Figure 5.

Table 1. The summarised lesson activities in the five lessons

Lesson	Lesson Activity
1	A – Practice on calculation of HCF and LCM
2	B – Real-life problems (paper mats and sweets)
3	C – Design a question and answer other group's question
4	D – Draw, present and show your work to the class
5	E – Solving problem-based learning question

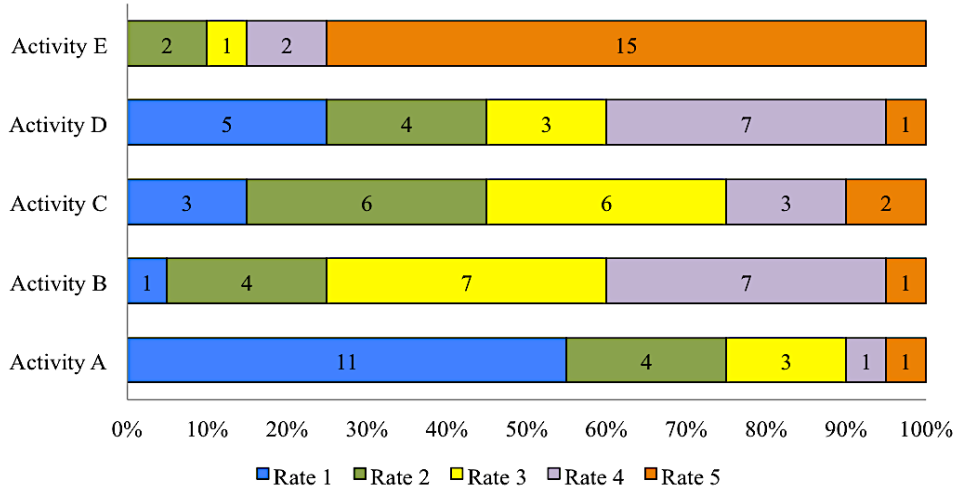


Figure 6. The students' rating for the five activities

The first activity (Activity A) from the first intervention lesson consisted of a calculation exercise on finding the HCF and LCM. As indicated in Figure 6, in total, 11 students (55%) chose this activity as their most favourite activity. From the reasons given, there were 6 students who stated that this activity was 'easy' and they were able to solve the problems without support from the teacher. Furthermore, they claimed that the questions given were understandable. The students who were not in favour of this activity were mainly confused between HCF and LCM.

The second activity (Activity B) involved practice on solving real-life problems with HCF and LCM. This activity posed quite a challenge on the students and it was clearly shown by their reactions to this lesson. Based on the students' responses, 2 students claimed that the activity was easy while 5 students had difficulties during this activity, 4 students had trouble understanding and 3 students were confused. One possibility of this occurrence may be due to the lack of exposure for these students to real-life situations involving HCF and LCM. Hence, this kind of activity, especially the questions given were new to them. Nevertheless, Students #15 and #17 enjoyed the activity as they stated that the activity was fun and easy. In addition, Student #15 stated that she could learn more skills, which meant that she had acquired new knowledge and new solving strategies during the intervention.

Activity C involved expressing their planning and creativity to produce their own question on HCF and LCM, with respect to real life situations in a group work setting, and subsequently, using this knowledge to solve other group's created question. For this activity, half of the class responded with positive feedback where they enjoyed creating the question and also answering other group's question. The same group of students also felt that the questions were easy. Student #4 expressed confidence in his individual capabilities and Student #15 believed that she could improve greatly in this topic. Similar to previous activities, there were still some students who had difficulties understanding and were confused regarding this topic.

The fourth activity (Activity D) also involved their creativity to re-image the received question from other groups and their solved answer on an A3 size poster for their presentation to the class. There were 13 students (65%) who gave favourable feedback, claiming that the activity was exciting, easy, interesting and increased their confidence. Student #8 thought that the purpose in conducting this particular activity was to show that they were able to solve the real-life problems. Meanwhile, Student #9 expressed that the reason they did the activity was so that they could share their knowledge, strategies and ideas to the class especially to those who lacked understanding in this topic. In contrast, there were only 2 students who claimed that this activity was difficult to understand and 1 student did not understand at all. There may be several reasons as to why this student did not understand, such as lacking of understanding towards the purpose of this activity, and another may be from stage fright in standing in front of the class and present a possible incorrect answer. Otherwise

this specific topic may be too advanced for the student at that time. It may also be due to lack of planning and creativity in answering and creating the questions.

The final activity (Activity E) of the intervention lesson involved solving the problem-based learning question. This activity was administered to promote higher-order-thinking. According to Figure 6, more than half the class (75%) rated this activity as their least favourite among the five activities. During the lesson, the teacher gave additional support in order to guide the students to the correct solution. Even though 11 students found this question difficult and they required additional time to grasp the question, yet some of the remaining students found this activity to be fun and easy.

In a subsequent question of the survey, the students were asked to recall the keywords that were related to both HCF and LCM. Almost all the students were correct and this showed that the students were able to recollect the keywords within the real-life worded problems. These could be attributed to the effectiveness of the intervention lessons, which enabled the students to not only gain some knowledge but to also understand and recall upon newly acquired knowledge. One student however wrote the term 'maximum' under the LCM part highlighting the confusion trends that were previously observed, or the student might have forgotten the term 'minimum' since she understood that the LCM involved the term 'smallest'. Another possible reason might be that the student did not fully understand the topic [20, 21] especially involving the real-life word problems. Despite that, the student was able to answer perfectly the keywords under HCF.

From the survey, the students were then asked to make suggestions on how to improve their solving capabilities with respect to real-life problems involving HCF and LCM. From the collated comments, there were some suggestions to improve the lessons by having frequent discussions. There are benefits from having frequent discussions in class such that the students could share more of their ideas and the teacher could provide better guidance to the students [17-19, 22]. There is also a possibility that more discussions could boost the students' performances when they solve real-life problems [8, 9, 17-19, 22-25], especially involving HCF and LCM.

4. Conclusions

Conducting the intervention lessons incited the students to strengthen their prior knowledge on HCF and LCM through discovering and experiencing real-life situation examples. These results were also supported by the procured survey questions in which the findings suggested that the students had difficulties in understanding the questions initially but after the intervention lessons, some students found the real-life problem questions were easy to solve. However, there were still a few students who needed more time to grasp the knowledge. Furthermore, some students also found the intervention lessons with the addition of activities to be enjoyable, fun and exciting in comparison to a normal class routine. Sufficient lesson plan preparation by the teacher was essential in providing suitable working exercises for the students. In addition to these exercises, the students were able to experience practical exercises during the intervention lessons and real-life problems in HCF and LCM. Referring back to Figure 6, more than half of the sampled students chose the first lesson with Activity A as their most and second most favourite lesson. This finding suggests that their preference in choosing the traditional drilling practice on the calculation of HCF and LCM, rather than choosing the activities involving real-life problems, was mainly due to the ease of completing the task activity. Importantly, the findings also indicated that most students favoured the group work approach because they were able to eventually share their ideas to the class with healthy competition learning style between each other or between groups. Grouped discussions helped the students to pose different strategies and applying group work helped them to plan and decide the best approach to solve the real-life problems. With the teacher's guidance, the students associated their strategies with their prior knowledge and understanding. Thus, these actions facilitated the students to be more aware of their real-life surrounding which they might relate the problem to HCF and LCM, and foster their metacognitive process to think creatively.

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